How to Measure the Nonlocal Phase of a Single Photon

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Abstract

The relative phase between spatially separated component waves of a single photon can be measured by joint interference with a second photon emitted by a known source. In the case of a single such phase (i.e. two component waves), the probability for a successful measurement is one half. This method can be implemented with current experimental techniques.

1 Introduction

Consider an ensemble of photons each separately impinging on the beamsplitter of Fig.1. Their state after leaving the beamsplitter and the phaseshifter on the right, is $|L\rangle + e^{i\theta}|R\rangle$, in an obvious notation. The simplest way of measuring the relative phase, θ, is to recombine the beams in some common region, and let them interfere. The phase is then determined in the usual way from the interference pattern. This is essentially the principle of telescopy. If the phase to be measured can take any value, a large number of measurements will be needed to determine it with precision. On the other hand, if we restrict the phase to either of two values differing by π, the two

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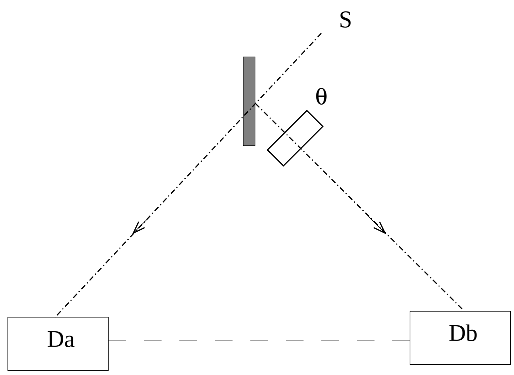

Figure 1: Measurement of nonlocal phase. Detectors $D_a,\ D_b$ act locally, but may share entanglement (Indicated by dashed line).

states are mutually orthogonal and an optimal measurement should be able to tell them apart every time, obviating the need to consider an ensemble. We do not restrict θ for now, for the sake of comparison with familiar single photon interference.

Suppose, however, that we limit ourselves to measurements performed locally on the two beams without recombining them. If the measuring devices share no quantum correlations (entanglement), we can gain no information on the relative phase. It is known, however, that if the two observers share an EPR pair this can be done, in principle, e.g., by placing an atom in each of the two remote locations, and having the photon absorbed by the one it comes in contact with. This is then followed by a Bell measurement on the two atoms[1].

As we shall see in the next section, the same goal can be accomplished much more simply, albeit with half the efficiency, by producing a second such photon, with known relative phase and recombining the two locally at each side as shown in Fig.3.

[1] After interacting with the atoms through the unitary transformation corresponding to absorption with probability one:

$$\left(|k_1\rangle + e^{i\phi}|k_2\rangle\right)|g\rangle_L|g\rangle_R \mapsto |0\rangle\left(|e_L, g_R\rangle + e^{i\phi}|g_L, e_R\rangle\right) \tag{1}$$

The two atoms are now in a superposition of two Bell states. Performing Bell measurements on the ensemble provides the phase. It has been shown[2], that Bell measurements on such a system can, in principle, be performed using local interactions and entanglement.

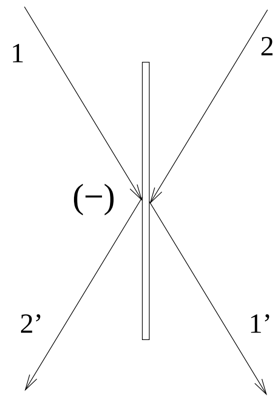

Figure 2: Beam splitter.

2 Two photon interference

Fig.2 depicts a beam splitter and two plane wave modes[2]. The beam splitter is assumed to be lossless and with coefficients of reflection and transmission of equal magnitude ('a 50-50 beam splitter'). The modes are symmetric with respect to reflection. There is still some freedom in the choice of the phases of these coefficients, and for definiteness we shall choose them to be real:

$$\begin{pmatrix} |1_{\mathbf{k}_1}\rangle \\ |1_{\mathbf{k}_2}\rangle \end{pmatrix} \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} |1_{\mathbf{k}'_1}\rangle \\ |1_{\mathbf{k}'_2}\rangle \end{pmatrix} \tag{2}$$

The interaction picture is implicitly assumed, so the only explicit time evolution is that caused by the beam splitters. Where we commit the common abuse of notation of denoting 'output modes', here distinguished by primes, where we really mean the same 'input modes' at a later time. This single photon scattering matrix is the same as the classical one. It also gives the full quantum scattering matrix in the Heisenberg representation[1]:

$$\begin{pmatrix} a^\dagger_{\mathbf{k}_1} \\ a^\dagger_{\mathbf{k}_2} \end{pmatrix} \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a^\dagger_{\mathbf{k}'_1} \\ a^\dagger_{\mathbf{k}'_2} \end{pmatrix} \tag{3}$$

[2]The output of a beam splitter of finite size cannot be described arbitrarily well by two plane waves (even when the input is such). At the end of this section we shall briefly outline the treatment of a more realistic state of the form $|L\rangle + e^{i\theta}|R\rangle$

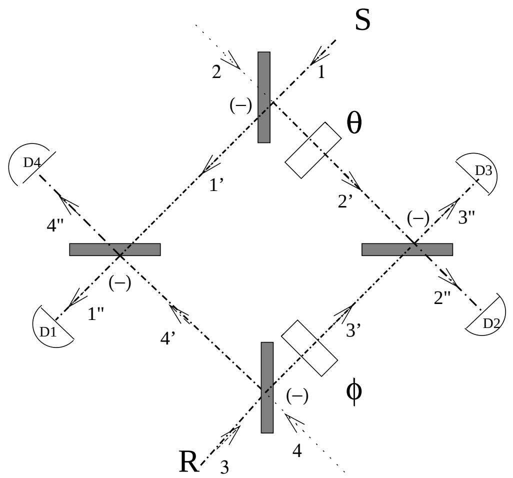

Figure 3: Set-up for detecting a photon's nonlocal phase by 2 photon interference.

The price we pay for the choice of real phases is an asymmetry, here in the reflection coefficients, between the left and right modes. This will be indicated in the figures by a minus sign on the side where the reflection is accompanied by a π phase shift.

Fig.3 depicts the interference pattern of the "source" photon of Fig.1 in mode 1, with a "reference" photon in mode 3. In other words, the input state is $|1_1, 1_3\rangle$, where we have dropped the **k**s. The resulting interference pattern can be calculated by applying Eq.3 once for each of the beam splitters, in the appropriate order to $|\text{in}\rangle = a_1^\dagger a_3^\dagger |0\rangle$:

$$\begin{aligned} a_1^\dagger a_3^\dagger &\mapsto \frac{1}{2}\left(a_{1'}^\dagger + e^{i\theta} a_{2'}^\dagger\right)\left(e^{i\phi} a_{3'}^\dagger + a_{4'}^\dagger\right) \\ &\mapsto \left(\frac{1}{2}\right)^2 \left[\left(a_{1''}^\dagger + a_{4''}\right) + e^{i\theta}\left(a_{2''}^\dagger - a_{3''}\right)\right]\left[e^{i\phi}\left(a_{2''}^\dagger + a_{3''}\right) + \left(-a_{1''}^\dagger + a_{4''}\right)\right] \\ &= \frac{1}{4}\left[\left(a_{4''}^{\dagger 2} - a_{1''}^{\dagger 2}\right) - e^{i(\theta+\phi)}\left(a_{3''}^{\dagger 2} - a_{2''}^{\dagger 2}\right)\right] \\ &+ \frac{e^{i\theta} + e^{i\phi}}{4}\left(a_{1''}^\dagger a_{3''}^\dagger + a_{2''}^\dagger a_{4''}^\dagger\right) + \frac{e^{i\theta} - e^{i\phi}}{4}\left(a_{1''}^\dagger a_{2''}^\dagger + a_{3''}^\dagger a_{4''}^\dagger\right). \end{aligned} \tag{4}$$

Or,

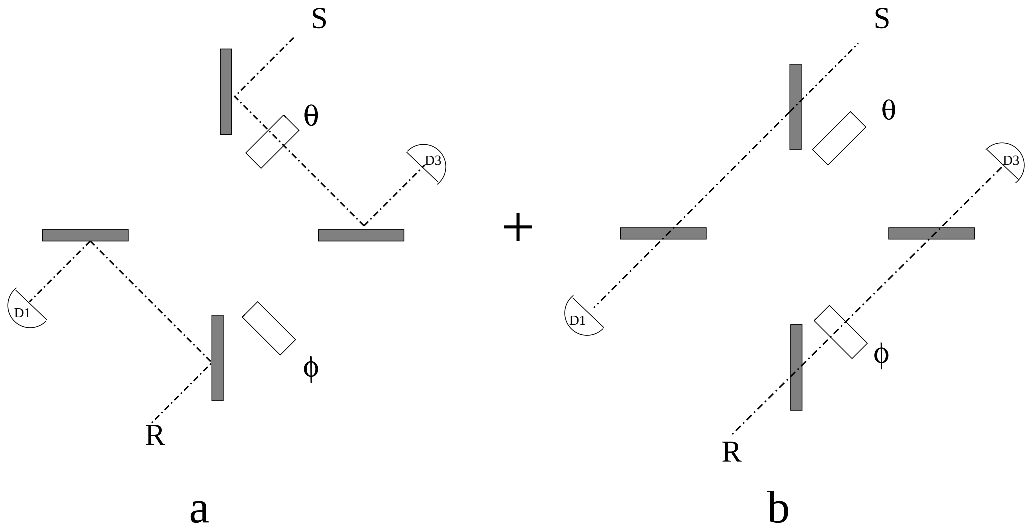

Figure 4: The two terms contributing to the output $|1_{1''}, 1_{3''}\rangle$.

$$
\begin{aligned}
|1_1, 1_3\rangle \quad &\mapsto \quad \frac{1}{\sqrt{8}} \left[|2_{4''}\rangle - |2_{1''}\rangle + e^{i(\theta+\phi)} \left(|2_{3''}\rangle - |2_{2''}\rangle \right) \right] \\
&+ \quad \frac{e^{i\theta} + e^{i\phi}}{4} \left(|1_{1''}, 1_{3''}\rangle + |1_{2''}, 1_{4''}\rangle \right) + \frac{e^{i\theta} - e^{i\phi}}{4} \left(|1_{1''}, 1_{2''}\rangle + |1_{3''}, 1_{4''}\rangle \right)
\end{aligned} \tag{5}
$$

Assuming perfectly efficient detectors, the probability of finding 2 photons at the same detector is[3] 1/2. This is the famous Hong-Ou-Mandel "bunching"[3]. These events give us no information on θ. The probability of getting clicks in either detectors 1 and 3, or 2 and 4; is $P(|1_1, 1_3\rangle) = \frac{1}{4}\cos^2\left(\frac{\theta-\phi}{2}\right) = P(|1_2, 1_4\rangle)$, and likewise $P(|1_1, 1_2\rangle) = P(|1_3, 1_4\rangle) = \frac{1}{4}\sin^2\left(\frac{\theta-\phi}{2}\right)$. The relative frequency of the occurrence of $\{\{1,2\},\{3,4\}\}$ vs. $\{\{1,3\},\{2,4\}\}$ evidently gives us information on θ. In particular, if θ can take on two values: $\theta \in \{\theta_0, \theta_0 + \pi\}$, we can choose $\phi = \theta_0$ and get perfect distinguishability when the 2 photons exit different beam splitters (i.e. half of the time).

[3]If the detectors do not distinguish between one and two photons, but are 100% efficient, this corresponds to having only one detector click.

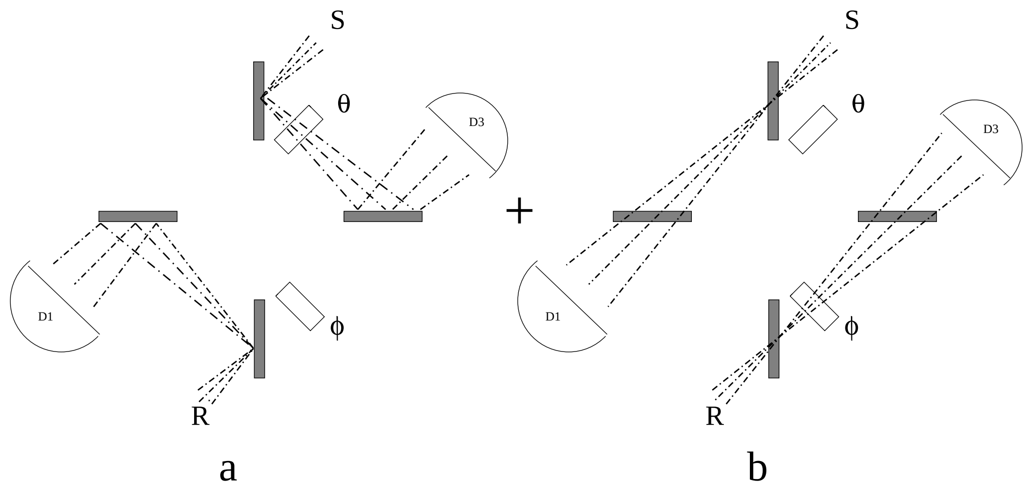

Figure 5: Two photon interference with non-planar waves. (Compare Fig.4)

Fig.4 illustrates graphically the two two-photon contributions to the probability amplitude, and hence to the probability of the outcome $|1_{1''}, 1_{3''}\rangle$:

$$P\left(|1_{1''}, 1_{3''}\rangle\right) = \left| e^{i\theta}\left(\frac{1}{\sqrt{2}}\right)^4 (-1)^2 + e^{i\phi}\left(\frac{1}{\sqrt{2}}\right)^4 \right|^2 = \frac{1}{4}\cos^2\left(\frac{\theta-\phi}{2}\right). \quad (6)$$

The rest of the terms can be interpreted similarly.

We are now ready to drop the assumption that $|L\rangle$, $|R\rangle$ are plane waves. As depicted in Fig.5, we consider the case where the primary and reference source each emit a superposition of several plane waves. It is still assumed that the two sources emit a beam with the same state, up to translation and rotation. The initial state is now $|\text{in}\rangle = \sum_i \alpha_i a^\dagger_{1i} \sum_j \beta_j a^\dagger_{3j}|0\rangle$. The figure shows graphically the geometrical optical argument demonstrating that the two pathways corresponding to the photons reaching a given pair of detectors are indeed indistinguishable and interfere.

3 Multiple relative phases.

The fact that we recover, on average, only half a bit of information on the local phase per photon, may seem troubling. Therefore, let us consider the generalization to more than one non-local phase, in which the situation is in some sense better.

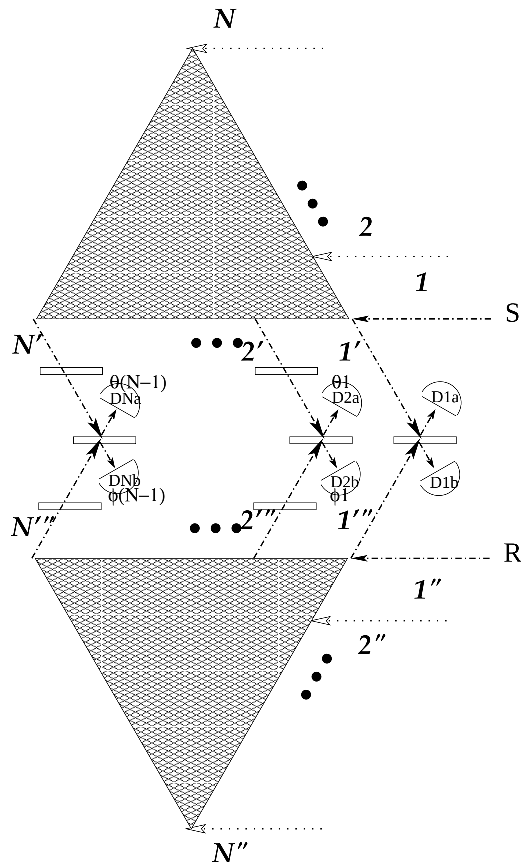

Figure 6: Two photon interference with N-1 independent relative phases. The large triangles denote symmetric N-mode multiport beam splitters[4].

Consider the states $|1_1\rangle + e^{i\theta_1}|1_2\rangle \ldots + e^{i\theta_{N-1}}|1_N\rangle$ with $\theta_i \in \{\theta_{i0}, \theta_{i0} + \pi\}$. The relative phase between each two terms again takes one of two values differing by π. If we now apply the analysis of the previous section to the set-up depicted in Fig. 6, and choose $\phi_i = \theta_{i0}$, we will find that with probability N^{-1} both photons will be absorbed by the same detector, giving away no information on any of the relative phases. With the complementary probability, we will get clicks at two detectors placed next to two different beam splitters, and the identity of these detectors will determine the relative phase of the two appropriate terms of the source state. Thus, we gain, on average, $1 - N^{-1}$ bits, where 'average' means expectancy. If we keep the same relative phases for different experimental runs, then as the number of runs becomes comparable to N, we shall get a lot of redundant information and the efficiency will go down. Relaxing the condition that the relative phases take on pairs of complimentary values is another possibility. We shall avoid these combinatorial complications by simply assuming that the phases are changed between experimental runs.

4 Discussion

As we have seen, by means of two photon interference, one can measure the nonlocal phase, or phases, between spatially separated components of a single photon. For N components, one can gain on average $1 - N^{-1}$ bits of information on the nonlocal phases per measurement. Recently, nonlocal measurements on a single photon using homodyne detection were discussed [6]. It seems to me that the simple scheme outlined above also sheds some light on the latter.

I have been careful not to use the controversial term 'the photon's wavefunction' in the hope of avoiding a long digression. A good discussion of this concept is found in [5].

As mentioned in the introduction, it is known that to measure a nonlocal property of a system by local devices, they must share entanglement. It is tempting in our present scheme to think of the photon of the reference source as the part of the measuring device carrying the entanglement. One might justify this distinction between the two photons by arguing that their orthogonal states make them distinguishable. From a quantum field point of view, a single nonlocal photon is indeed an entangled state (between of the field at various locations). As long as the two photons don't overlap, we can

think of the overall state as a product of the two single-photon states. The only places where the two photons overlap, are precisely in the vicinity of the local detectors.

Finally, note the similarity to Hanbury-Brown–Twiss effect[7]. Here, however, one of the interfering photons comes from a reference source.

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